RADICALS FOR THE FRESHMAN.

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The course in radicals, here outlined, does not pretend to offer anything new or startling. It merely suggests one way in which the freshman may be led to walk with some degree of confidence through a subject which is apt to seem to him a blind and useless maze of algebraic trickery. The work required of him is limited to what may be necessary in solving and testing equations of the quadratic form, in solving simple radical equations, and in evaluating formulæ.

Before attacking the subject of irrational quantities, the freshman reviews the definition of and conceptions concerning the root of a quantity, taking the square or cube roots of small arithmetic numbers which are perfect squares or cubes respectively, and reviewing the exponent rule for taking the root of a monomial, which was previously evolved in connection with factoring. Throughout this course, by the way, the term "root" and not "principal root" is used, for no mention is made of the possible imaginary roots.

He then passes quickly to the square root of polynomial perfect squares. This work he does primarily as a basis for taking the square root of numbers. He is led to see that an arithmetic number may be treated as a polynomial, so that the very rule used in algebra may be made to apply in arithmetic. He acquires proficiency in this work by finding the lengths of the sides of right triangles, diagonals of rectangles, sides of squares, whose areas are given, etc., the given numbers being carefully expurgated, at this stage, by means of the $3:4:5$ relation between the sides of a right triangle, so that he need take the square roots of perfect squares only.

Exponents other than positive integers come next for his consideration. Assuming that the exponent rules already learned always hold true, he develops meanings for the fractional zero, and negative exponents, agreeing with that assump-

tion. He has a little practice in the use of these exponents, principally because it simplifies the work in surds that follows.

Up to this point, he has considered rational quantities only. He now comes to irrational expressions. Would they had a more inviting name! They present difficulties enough, without being thus branded.

To begin, he considers some simple surd, $\sqrt{2}$ for instance. "There is no such thing," he says. Thereupon, he is led back to the right triangle, with which he has worked before. Using *one* for each leg, he finds the hypotenuse to be $\sqrt{2}$. Given varied values for the legs, he finds a number of surds representing lengths of lines, which soon convinces him that a surd *may be,* at any rate, a perfectly definite quantity and is therefor not nearly so absurd as it sounds. He then finds the approximate value of $\sqrt{2}$, as a decimal. "If it is a perfectly definite quantity, why can it not be expressed definitely, as a decimal?" In answer, he is told to evaluate the common fraction $1/3$ as a decimal. He soon decides that the difficulty is not with the surd or common fraction, but lies in the limitations of the decimal system. If, then, a surd is a definite quantity, which may appear in his formulæ and quadratic equations, he must know how to treat it.

First, he encounters the three ways in which a radical may be simplified, to facilitate handling it, and to these he gives a large part of the time and attention he puts on radicals.

From his previous study of the meaning of fractional exponents, he writes $\sqrt{ab}=(ab)^{\frac{1}{2}}=a^{\frac{1}{2}}b^{\frac{1}{2}}=\sqrt{a}\ \sqrt{b}$. Given $\sqrt{36}$, he says at once, "6." Can 36 be divided into two factors, whose roots might be taken separately, as in the preceding illustration? Yes. $\sqrt{9}\cdot\sqrt{4}=3.2=6$, the same result as before. He is then given a radicand having only one perfect square factor

$$\sqrt{50}=\sqrt{25}\cdot\sqrt{2}=5\sqrt{2}.$$

One freshman is set to work to find the approximate value of $\sqrt{50}$, while another finds the value of $5\sqrt{2}$,—results, the same. "What is the use of simplifying, when the approximate value may be obtained as easily, without simplifying?" Suppose a number of lines, represented by surds, were to be computed, as,

for instance, $\sqrt{12}$, $\sqrt{27}$, $\sqrt{48}$, $\sqrt{75}$, etc. There are four square root examples to be done. By simplifying each, $2\sqrt{3}$, $3\sqrt{3}$, $4\sqrt{3}$, $5\sqrt{3}$, one square root example results and the work is done in about one third the time. Other reasons for simplifying become obvious as he proceeds. This work is limited to the square and cube roots and simpler cases of higher degree. To increase his efficiency in handling this first method of simplifying radicals, he learns the squares of numbers up to 25 and the cubes to 10. He is also encouraged but not required to learn the decimal values for $\sqrt{2}$ and $\sqrt{3}$.

In attacking radicals with fractional radicands, he is guided somewhat after this fashion:

$$\frac{2}{5}\cdot\frac{2}{5}=\frac{4}{25}. \quad \therefore \sqrt{\frac{4}{25}}=\frac{2}{5} \text{ or } \frac{\sqrt{4}}{\sqrt{25}};$$

$$\therefore \sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{\sqrt{4}}=\frac{\sqrt{3}}{2}=\frac{1.732}{2}=.866;$$

$$\sqrt{\frac{2}{3}}=\frac{\sqrt{2}}{\sqrt{3}}=\frac{1.414}{1.732}.$$

This division is performed. Now, by simplifying first thus

$$\sqrt{\frac{2}{3}\cdot\frac{3}{3}}=\sqrt{\frac{1}{9}}\cdot\sqrt{6}=\frac{\sqrt{6}}{3}=\frac{2.449}{3},$$

the last operation can be done mentally. It takes only two or three such instances to convince him of the advantage of removing the fraction from under the radical sign, before computing. After the first few examples, he does the work of simplifying under one radical sign.

He next has a very little work in lowering the order of a surd, first writing the expressions with fractional exponents and reducing the fractions to lowest terms; later, when the principle is clear, dividing the index and exponents by any common factor.

He applies these methods of simplifying radicals in evaluating formulæ such as:

$b = \sqrt{ae}$, find the value of b,
$c = \sqrt{a^2 + b^2}$, find value of c,
$A = \sqrt{s(s-a)(s-b)(s-c)}$, find value of A,
$P = 2\pi\sqrt{\frac{h}{g}}$, find value of P,

etc., writing results in simplest form first, then finding the decimal values of a few.

Then he solves incomplete and complete quadratic equations, involving radicals, such as:

$$4A^2 = 7, \text{ solve for } A,$$
$$A = \pi R^2, \text{ solve for } R,$$
$$s = \tfrac{1}{2}gt^2, \text{ solve for } t,$$
$$W = \frac{Q}{2c}, \text{ solve for } Q,$$
$$d = vt + 16t^2, \text{ solve for } t,$$
$$R = \frac{s^2 + h^2}{2h}, \text{ solve for } h,$$

etc., and problems involving formulæ and equations of the same type. The following will suffice for illustration.

1. How long will it take a body to fall 1,280 ft.? ($s = 16t^2$.)

2. The foreman of a shop reads in his book of instructions that the safe load (l), in pounds, that can be hoisted by a rope, c-inches in circumference, is found by the formula, $l = 100c^2$. How big round must his rope be to lift 500 lbs. safely?

3. A tinsmith wishes to make some cylindrical gallon cans. They are to be ten inches high. What radius must he use in drawing the base, if $V_c = \pi R^2 h$ and 1 gal. = 231 cu. in.

4. The horse power of a gasoline engine is computed by the formula

$$H = \frac{D^2 N}{2.5},$$

where H = horse power, D diam. of cylinder in inches, N = number of cylinders.

In an engine of one cylinder, find the diameter of the cylinder required to yield 40 H. P.

He does a little work in addition and subtraction, to be used later in connection with multiplication, radical equations, and checking work in quadratic equations.

He works a little with multiplication and division of radicals of the same order or of differing ones of low degrees and less still with involution and evolution. The principles are developed by means of fractional exponents. In practice, however, he does most division by rationalization, to which he gives rather more attention, limiting the work, however, to fractions having, as denominators, monomial surds of the second or third degree or binomial surds of the second. To encourage rationalization before computing the values of fractions as decimals, he is given a few fractions which are greatly simplified thereby and is required to compute them in both ways.

This work is applied in such formulæ as

$s = R\sqrt{3}$, solve for R correct to 2 places,
$h = \frac{1}{3} b\sqrt{3}$, solve for b,
$a = \frac{1}{4} s^2\sqrt{3}$, solve for s,
$s = 2a(\sqrt{2} - 1)$, solve for a,

etc.

The freshman finishes his work in surds by solving simple radical equations, testing results to be sure he has not introduced roots by squaring. He is shown the possibility of that by some such simple device as this. If $x = 2$, squaring $x^2 = 4$, writing the second equation $x^2 - 4 = 0$ or

$$(x + 2)(x - 2) = 0,\ x = 2 \text{ or } -2.$$

Throughout the course the aim is to make the freshman feel that any algebraic manipulation required of him is the shortest road to a desired goal, and not merely a devious path for the sake of the journey.

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